Topic 9: Calculus Option	*Limits of Sequences and Functions*

Algebraic limit laws:	$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$	$\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
	$\lim_{x\to a}[f(x)\times g(x)] = \lim_{x\to a} f(x) \times \lim_{x\to a} g(x)$	$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
	$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$	

L'Hopital's rule states if the following occurs:			$\frac{1}{0^-} = -\infty$	$\frac{1}{0} = Undefined$	$\frac{1}{0^+} = \infty$
$\mathbf{\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}}$	or	$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$			
Differentiate top and bottom fraction separately		$\lim_{x\to a}\frac{f(x)}{g(x)} \overset{L'H}{=} \lim_{x\to a}\frac{f'(x)}{g'(x)}$	$\frac{1}{\infty} = 0$	$\frac{\infty}{\infty} = Undefined$	$\frac{0}{0} = Undefined$

The Squeeze Theorem states if: *Use when involving trigonometric functions*			Interval Notation:	A closed interval $[a,b]$ includes end points	An open interval (a,b) excludes end points
$\boldsymbol{g(x) \le f(x) \le h(x)}$	and	$\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$	A function is continuous at $x = a$ when: $\lim_{x\to a^-} f(x) = \lim_{x\to a} f(x) = \lim_{x\to a^+} f(x)$		
Then:	$\lim_{x\to a} f(x) = L$				

The Absolute Value Theorem states:	if $\lim_{n\to\infty}\|a_n\| = 0$, then $\lim_{n\to\infty} a_n = 0$
A function is differentiable at a point $\boldsymbol{x_0}$ if:	$f(x)$ must be continuous at x_0
	$f(x)$ must not have a "sharp point" at x_0
	the tangent to $f(x)$ at x_0 must be vertical

The derivative of a function $\boldsymbol{f(x)}$ at the point $\boldsymbol{x_0}$ can be found if the limit exists by:
If not, then the function is not differentiable at that point

$\boldsymbol{f'(x_0) = \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}}$	or	$f'(x_0) = \lim_{h\to 0}\frac{f(x)-f(x_0)}{x-x_0}$

Rolle's Theorem requires three conditions: *a and b are found from the given interval*	$f(x)$ is continuous on $[a,b]$	Mean Value Theorem requires three conditions: *a and b are found from the given interval*	$f(x)$ is continuous on $[a,b]$
	$f(x)$ is differentiable on (a,b)		$f(x)$ is differentiable on $]a,b[$
	$f(a)$= $f(b)$		$c \in]a,b[$
If three conditions are met then there must be a point $\boldsymbol{c}$ such that $\boldsymbol{f'(c) = 0}$		If these conditions are met, then there exists a point c such that: $f'(c) = \frac{f(b)-f(a)}{b-a}$	

Topic 9: Calculus Option	*Improper Integrals*

The Fundamental Theorem of Calculus states if $\boldsymbol{f(x)}$ is continuous on $[\boldsymbol{a}, \boldsymbol{b}]$ where:		
$\boldsymbol{F(x) = \int_a^x f(t)\,dt \quad a \leq x \leq b}$	*then*	$F'(x) = f(x)$
And for any function where $\boldsymbol{g(x)}$ such that $\boldsymbol{F'(x) = f(x)}$		
$\boldsymbol{\int_a^b f(x)\,dx = F(b) - F(a)}$	*Where $F(x)$ is the antiderivative of $f(x)$*	

The p-series states for: $\int_1^\infty \frac{1}{x^p}\,dx$	Converges if $p > 1$
	Diverges if $p \leq 1$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$	$\int_a^b = -\int_b^a$

Integrals in the form $\int_a^\infty \boldsymbol{f(x)\,dx}$ are known as improper integrals Improper integrals are either convergent or divergent:	Convergent when $\int_a^\infty f(x)\,dx = 0 \text{ or } finite\ value$
	Divergent when $\int_a^\infty f(x)\,dx = \pm\infty$

When integrating improper integrals, use $\lim\limits_{t\to\infty}$ and replace ∞ with $\boldsymbol{t}$	$\int_a^\infty f(x)\,dx = \lim\limits_{t\to\infty} \int_a^t f(x)\,dx =$

The Comparison Test for improper Integrals states:	If $0 \leq f(x) \leq g(x)$ for all $x \geq a$ then:
	$\int_a^\infty f(x)\,dx$ is convergent if $\int_a^\infty g(x)\,dx$ is convergent
	$\int_a^\infty g(x)\,dx$ is divergent if $\int_a^\infty f(x)\,dx$ is divergent

The Rienmann Sum states:	or a decreasing function $f(x)$ for all $x > a$, then there is an upper and lower sum such that:	$\sum\limits_{k=a+1}^{\infty} f(k) < \int_a^\infty f(x)\,dx < \sum\limits_{k=a}^{\infty} f(k)$
	For an increasing function $g(x)$ for all $x > a$, then there is an upper and lower sum such that:	$\sum\limits_{k=a}^{\infty} g(k) < \int_a^\infty g(x)\,dx < \sum\limits_{k=a+1}^{\infty} g(k)$

Topic 9: Calculus Option		*Series Part 1*			
A series consists of $\boldsymbol{a_1} + \boldsymbol{a_2} + \boldsymbol{a_3}$ ...		$\sum_{n=1}^{\infty} a_n = S$	$\sum_{k=1}^{\infty} a_k = S_n$		
It is denoted by:		For a total sum	For a partial sum		
The Divergence Test states:		If $\lim_{n\to\infty} a_n \neq 0$ or does not exist then $\sum a_n$ diverges			
		If $\lim_{n\to\infty} a_n = 0$ then $\sum a_n$ may converge or may diverge			
Key series types:	Geometric Infinite Series: $\sum_{n=1}^{\infty} ar^{n-1}$	Converges if $\|r\| < 1$	Diverges if $\|r\| \geq 1$		
		Sum of infinite geometric series:	$= \frac{a}{1-r}$		
	Telescoping Series: *Partial Fractions*	Find S_n equation	Simplify S_n	Take $\lim_{n\to\infty} S_n$	
	P-Series: $\frac{1}{n^p}$ (*Harmonic Series:* $\frac{1}{n}$)	Converges if $p > 1$			
		Diverges if $p \leq 1$			
The Comparison Test states given for two series of positive terms:		If $a_n \leq b_n$ for all of n and $\sum b_n$ converges, then $\sum a_n$ also converges			
		If $a_n \geq b_n$ for all of n and $\sum b_n$ diverges, then $\sum a_n$ also diverges			
Steps:		1. Check if positive ($0 \leq a_n$)			
		2. Find b_n (b_n generally is a_n take away a useless term)			
		3. Determine whether b_n is smaller or larger than a_n			
		3. Find if b_n is convergent or divergent			
The Limit Comparison Test States:		If $\sum \frac{a_n}{b_n}$ exists and a_n and b_n are positive to use limit comparison test:	Then both series either converges of diverges		
		Steps:	1. Find b_n (b_n generally is a_n take away a useless term)		
			2. Find if a_n and b_n are positive and if $\sum \frac{a_n}{b_n}$ exists		
The Integral Test states for $\boldsymbol{f(n) = a_n}$, if		then	$\sum_{n=1}^{\infty} a_n$	$\int_{n=1}^{\infty} f(x)dx$	Will both converge or diverge
Positive	Decreasing	Continuous	1. Check criteria: If it's positive decreasing and continuous	2. Then integrate a_n	

Topic 9: Calculus Option	*Series Part 2*
The Alternating Series Test states for: $\sum_{n=1}^{\infty}(-\mathbf{1})^n \boldsymbol{a_n}$	if $\lim_{n\to\infty}\lvert a_n\rvert = 0$ and $\lvert a_{n+1}\rvert < \lvert a_n\rvert$, then the series will converge
If: $\sum \boldsymbol{a_n}$ converges, then $\lim_{n\to\infty} \boldsymbol{S_n} = \boldsymbol{S}$ However, if $\boldsymbol{n}$ does not approach ∞ then the Error of a Sum can be found:	The Absolute Converge states: For $\sum a_n$, if $\sum\lvert a_n\rvert = \lvert a_1\rvert + \lvert a_2\rvert + \cdots$ is convergent, then $\sum a_n$ is absolutely convergent *Use if series is not always positive, or not alternating, or when dealing with trig* 1. Write $\sum a_n$ as $\sum\lvert a_n\rvert$
For alternative series only: $\lvert \boldsymbol{R_n}\rvert = \lvert \boldsymbol{S} - \boldsymbol{S_n}\rvert \leq \boldsymbol{a_{n+1}}$ $\lvert Error\rvert \leq \lvert \boldsymbol{a_{n+1}}\rvert$ $\boldsymbol{a_{n+1} \leq error\ specified}$	Note: Absolute convergence, is stronger than convergence If a series is absolute convergent, it must be convergent Conditional convergence is when series converges, but is not absolutely convergent
The ratio test stages if: *Don't need to use divergent test for ratio test*	1. $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1$, $\sum a_n$ is absolutely convergent 2. $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert > 1$, $\sum a_n$ is divergent 3. $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert = 1$, $\sum a_n$ is inconclusive