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| Topic 9: Calculus Option | | | | | | | | | | | | | *Limits of Sequences and Functions* | | | | | | | | | | | | |
| Algebraic limit laws: |  | | | | | | | | | | | | | | |  | | | | | | | | | |
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| L’Hopital’s rule states if the following occurs: | | | | | | | | | | | | | | | | | |  | |  | | | |  | |
|  | | or | | | |  | | | | | | | | | | | |
| Differentiate top and bottom fraction separately | | | |  | | | | | | | | | | | | | |  | |  | | | |  | |
| The Squeeze Theorem states if:  *Use when involving trigonometric functions* | | | | | | | | | | | | | | | | | | Interval Notation: | | | A closed interval includes end points | | | | An open interval excludes end points |
|  | | | | | | | and | | |  | | | | | | | | A function is continuous at when: | | | | | | | |
| Then: |  | | | | | | | | | | | | | | | | |
| The Absolute Value Theorem states: | | | | | | | | | | | | if , then | | | | | | | | | | | | | |
| A function is differentiable at a point if: | | | | | | | | | | | | must be continuous at | | | | | | | | | | | | | |
| must not have a “sharp point” at | | | | | | | | | | | | | |
| the tangent to at must be vertical | | | | | | | | | | | | | |
| The derivative of a function at the point can be found if the limit exists by:  *If not, then the function is not differentiable at that point* | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | | | | | | | | | | | or | | | |  | | | | | | | | | | |
| Rolle’s Theorem requires three conditions:  *a and b are found from the given interval* | | | is continuous on | | | | | | | | | Mean Value Theorem requires three conditions:  *a and b are found from the given interval* | | | | | | | | | | is continuous on | | | |
| is differentiable on | | | | | | | | | is differentiable on | | | |
| = | | | | | | | | |  | | | |
| If three conditions are met then there must be a point such that | | | | | | | | | | | | If these conditions are met, then there exists a point such that: | | | | | | | | | | | | | |
| Topic 9: Calculus Option | | | | | | | | | | | | | | *Improper Integrals* | | | | | | | | | | | |
| The Fundamental Theorem of Calculus states if is continuous on where: | | | | | | | | | | | | | | The p-series states for: | | | | | | | | | Converges if | | |
|  | | | | | *then* | | | |  | | | | | Diverges if | | |
| And for any function where such that | | | | | | | | | | | | | |  | | | | | | | | | |  | |
|  | | | | | | | | *Where is the antiderivative of* | | | | | |
| Integrals in the form are known as improper integrals  Improper integrals are either convergent or divergent: | | | | | | | | | | | | | | Convergent when | | | | | | | | | | | |
| *Divergent when* | | | | | | | | | | | |
| When integrating improper integrals, use and replace with | | | | | | | | | | | | | | | | |  | | | | | | | | |
| The Comparison Test for improper Integrals states: | | | | | If for all then: | | | | | | | | | | | | | | | | | | | | |
| is convergent if is convergent | | | | | | | | | | | | | | | | | | | | |
| is divergent if is divergent | | | | | | | | | | | | | | | | | | | | |
| The Rienmann Sum states: | | | | | or a decreasing function for all, then there is an upper and lower sum such that: | | | | | | | | | | | | | |  | | | | | | |
| For an increasing function for all , then there is an upper and lower sum such that: | | | | | | | | | | | | | |  | | | | | | |

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| Topic 9: Calculus Option | | | | | | | *Series Part 1* | | | | | | |
| A series consists of | | | |  | | | | |  | | | | |
| It is denoted by: | | | | For a total sum | | | | | For a partial sum | | | | |
| The Divergence Test states: | | | | If or does not exist then diverges | | | | | | | | | |
| If then may converge or may diverge | | | | | | | | | |
| Key series types: | | Geometric Infinite Series: | | Converges if | | | | | Diverges if | | | | |
| Sum of infinite geometric series: | | | | |  | | | | |
| Telescoping Series:  *Partial Fractions* | | Find equation | | | | Simplify | | | | Take | |
| P-Series:  (*Harmonic Series:* ) | | Converges if | | | | | | | | | |
| Diverges if | | | | | | | | | |
| The Comparison Test states given for two series of positive terms: | | | | If for all of and converges, then also converges | | | | | | | | | |
| If for all of and diverges, then also diverges | | | | | | | | | |
| Steps: | | | | 1. Check if positive () | | | | | | | | | |
| 2. Find ( generally is take away a useless term) | | | | | | | | | |
| 3. Determine whether is smaller or larger than | | | | | | | | | |
| 3. Find if is convergent or divergent | | | | | | | | | |
| The Limit Comparison Test States: | | | | If exists and and are positive to use limit comparison test: | | | | | | Then both series either converges of diverges | | | |
| Steps: | 1. Find ( generally is take away a useless term) | | | | | | | | |
| 2. Find if and are positive and if exists | | | | | | | | |
| The Integral Test states for , if | | | | then | |  | | | | |  | | Will both converge or diverge |
| Positive | Decreasing | | Continuous | 1. Check criteria: If it’s positive decreasing and continuous | | | | | | | 2. Then integrate | | |

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| Topic 9: Calculus Option | | | *Series Part 2* | |
| The Alternating Series Test states for: | if and , then the series will converge | | | |
| If: converges, then  However, if does not approach then the Error of a Sum can be found: | The Absolute Converge states: For , if is convergent, then is absolutely convergent | | | *Use if series is not always positive, or not alternating, or when dealing with trig* |
| 1. Write as | | | |
| For alternative series only: | Note:  Absolute convergence, is stronger than convergence  If a series is absolute convergent, it must be convergent  Conditional convergence is when series converges, but is not absolutely convergent | | | |
| The ratio test stages if:  *Don’t need to use divergent test for ratio test* | | 1. , is absolutely convergent | | |
| 2. , is divergent | | |
| 3. , is inconclusive | | |