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| Topic 2: Functions and Equations | *Polynomials* |
| The Remainder Theorem states:  | The Factor Theorem states: |
| If a polynomial $f(x)$ is divided by $x-k$, then$$remainder=f(k)$$ | A polynomial $f(x)$ has a factor $ (x-k)$ if and only if :$$ f\left(k\right)=0$$ |
| Polynomial function: Factors, Roots, Zeros $$y=x^{2}+2x-15$$ | Factors are: $(x+5)$ and $(x-3)$ | The line of symmetry of$ y=ax^{2}+bx-c$ is: $x=\frac{-b}{2a}$This can also be used to find turning point of quadratic by plugging $x$ |
| Zeros are: $-5$ and $3$ |
| X-Intercepts are at: $-5$ or $-3$ | The number of solutions of a quadratic equation depends on the value of the discriminant: | $$∆=b^{2}-4ac$$ |
| Roots/Solutions are: $x=5$ or $3$ | $$∆>0$$*2 Real distinct solutions* | $$∆=0$$*One Real Solution* | $$∆<0$$*No real solutions* |
| Topic 2: Functions and Equations | *The Theory of Functions* |
| Function: A set of ordered pairs in which every x-value has a unique y-value. |
| In order to be a function, the graph of an equation must pass the vertical and horizontal line test |
| The Vertical Line Test States: | A relation is a function if a vertical line intersects the graph of a relation at only one point,  |
| The Horizontal Line Test States: | A function is a one-to-one function if a horizontal line crosses the graph onceOtherwise, it is a many-to-one function |
| Rationale Functions are a ratio of two polynomials: | Asymptote & intercepts of a rational function: | Vertical Asymptote: $ VA=-\frac{d}{c}$ *(where* $y$ *is impossible, thus* $denominator=0$*)* |
| Horizontal Asymptote: $ HA$ | $$deg\left(num\right)=deg⁡(den)\rightarrow $$ | $=\frac{a}{c}$ *(substitute* $\infty $ *for* $x$*)* |
| $$deg\left(num\right)<deg⁡(den)\rightarrow $$ | $$=0$$ |
| $$deg\left(num\right)>deg⁡(den)\rightarrow $$ | $$=none$$ |
| $$f\left(x\right)=\frac{ax+b}{cx+d}$$ | X-intercept: $ x=-\frac{b}{a}$ *(where* $y=0$*)* |
| Y-intercept: $ y=\frac{b}{d}$ *(where*$ x=0$*)* |
| Interval Notation | Set Builder Notation | A function is odd when:$ f\left(-x\right)=-f(x)$ |
|  |  | A function is even when:$ f\left(-x\right)=f(x)$ |
| Inverse functions:$$f^{-1}(x)$$ | Reflection of$ f(x)$ on the line $ y=x$ |
| Swaps domain and range of $f(x)$ |
| $$f(f^{-1}(x))=f(x)$$ |
| Topic 2: Functions and Equations | *Transformations of Graphs* |
| Shifts | $y=f\left(x-h\right)$ shifts $y=f(x)$ to the right by $h$ units |
| $y=f\left(x+h\right)$ shifts $y=f(x)$ to the left by $h$ units |
| $y=f\left(x\right)+k$ shifts $y=f(x)$ up by $h$ units |
| $y=f\left(x\right)-k$ shifts $y=f(x)$ down by $h$ units |
| Reflections | $y=f\left(-x\right)$ reflects $y=f(x)$ across the y-axis |
| $y=-f\left(x\right)$ reflects $y=f(x)$ across the x-axis |
| Stretches | If$ a>1$, transformation is a stretch | If $a<1$, transformation is a compress |
| $y=f\left(ax\right)$ stretches/compresses $y=f(x)$ horizontally, by $\frac{1}{a}$ |
| $y=af\left(x\right)$ stretches/compresses $y=f(x)$ vertically, by $a$ |
| Modulus | $$\left|f(x)\right|$$ | Turns all x values positive |
| $$f(\left|x\right|)$$ | Reflects the graph to the right of the y-axis in the y-axisIgnore the left hand side part of the graph |
| $$\frac{1}{f(x)}$$ | Zeros of $f(x)$ (when they exist) are the vertical asymptotes of$ \frac{1}{f(x)}$ | Zeros of $\frac{1}{f(x)}$ are the vertical asymptotes of$ f(x)$ |
| If $c$ the y-intercept of$ f(x)$, then $ \frac{1}{c}$ is the y-intercept of $\frac{1}{f(x)}$ |
| The minimum value of $f(x)$ is the maximum of $\frac{1}{f(x)}$ | The minimum value of $\frac{1}{f(x)}$ is the maximum of $f(x)$ |
| When $f\left(x\right)>0$, $\frac{1}{f(x)}>0$ | When $f\left(x\right)<0$, $\frac{1}{f(x)}<0$ |
| When $f(x)$ approaches 0, $\frac{1}{f(x)}$ will approach $\pm \infty $ | When $f(x)$ approaches $\pm \infty $, $\frac{1}{f(x)}$ approaches 0 |